1591 Villa Rotunda
Vicenza, Italy

Andreas Palladio

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Introduction

The use of proportions in the natural world such as those from measurements of the human body or musical interval in the expression of harmony in the universe became very common in the field of art and architecture during the Renaissance period (Wittkower 24). As a Renaissance architect, Andreas Palladio established rules of proportions on room dimensions based on ancient Greek, Roman and Renaissance mathematics for the creation of buildings that were intended to impose the experience of harmony into their viewers and users. However, because of the lack of information regarding the rationale for the selection of these proportions, it remains uncertain whether they are merely mathematical representation of the humanist ideal, or whether they hold a deeper purpose in affecting the functional aspect or human experience in the rooms or the building as a whole.

This research paper addresses this uncertainty by investigating Palladio’s rules of proportions and their application to his exemplary built work – the Villa Rotunda. The investigation is then followed by a lighting study to examine how the proportions of selected rooms built in the Villa Rotunda affect illumination of interior space. It is hypothesized that the rules of proportion applied by Andreas Palladio in the design of his buildings were not only mere reflections of the mathematical ideal, but they were established to optimize the human experience, such as the perception of light, in the building.

Pursuit of the Ideal in the Renaissance

The Renaissance has also been referred to as the Age of Humanism, as artists and architects pursued the creation of the ideal form by following mathematical rules embedded in the natural forms, such as the human body. This is because for many Renaissance scholars and artists, “the human body derive all measures and their denominations and in it is to be found all and every ratio and proportion by which God reveals the innermost secrets of nature” (Wittkower 25). This emerging movement to link art and mathematics was accelerated by the need for craftsmen at this period to provide mathematical foundation to areas of art and architecture, in order to raise them from the level of the mechanical to that of the liberal arts and science (Wittkower 113).
Under this movement, proportion has become a critical realm explored by Renaissance artists, and many works of art and buildings at the period demonstrates “[c]orrespondence between perfect numbers, proportions of the human figure and elements of musical harmony” (Rowe 8). Renaissance architect Alberti, a predecessor of Palladio also emphasized the significance of the ideal proportion “as a projection of the harmony of the universe” (Wittkower 25).

Andreas Palladio was born in the midst of this cultural development, which has shaped his attitude towards architecture and mathematics.

Proportion by Palladio

Born in Padua on November 30, 1508, Andrea Palladio, originally named Andrea Di Peitro Dalla Gondola, was one of the most influential Italian Renaissance Architect. His early experience as a stonemason in Padua and Vicenza has fostered his artisan skills and sensibility to building material (Wassell). In his early thirties, Palladio talent was first recognized by classical humanist scholar Count Gingiorgio Trissino, who introduced him to the study of architecture in Rome, focusing on the study of classics and Vitruvius’ principles in architecture (Mitrovic 18). This has strengthened Palladio’s fluency in the classical architectural language, demonstrated by his sensible use of symmetry and classical orders in his early works (Wassell 21).

Palladio practiced professionally in Vicenza and the Northern Italy and collaborated closely with other Humanists and Mathematicians at the time, such as, Daniele Barbaro and Silvio Belli, who reinforced his sensitivity to harmony in geometry and proportions (Padovan 24). Palladio’s classical vocabulary developed extensively between late 1540s and 1550s. His work during this time does not only demonstrate his mastery of geometry and the Roman Classical style, but more importantly his inventiveness in assembling the elements of his vocabulary “to
shape space, articulate solid and modulate light with elegance, grace and beauty” (Wassell 19).

In 1570, he published his treatise, I quattro libri dell’architettura, which detailed his principles in creating classical architecture. For some parts, it was a further articulation of the ideas expressed in the previous treatises by Vitruvius and Alberti (Wittkower 25). Palladio presented also his key design philosophies in substantial detail, one of which was his rules of ratio and proportion (Wassell 20).

Visible in his dimensioned plans in his Quattro Libri, Palladio placed a strong emphasis on proportions, as he strove to achieve unification of the plan, elevation and section in his work (Padovan 30). He stated:

“Beauty will derive from a graceful shape and the relationship of the whole to the parts, and of the parts among themselves and to the whole, because buildings must appear to be like complete and well-defined bodies, of which one member matches another and all the members are necessary for what is required.” (Palladio 6-7)

Palladio’s new approach of unification and integration of part to the whole was strongly influenced by his close collaboration with humanist and mathematician Daniele Barbaro. His establishment of rules for proportion was based on precedents set by Vitruvius and Alberti, as well as the development of Renaissance mathematics. In the Quattro Libri, Palladio has set rules for ratios of plan dimensions, as well as for room height that are intended to achieve the most aesthetically pleasing proportions (Wassell 31)

Proportions in Plan

In the Quattro Libri, Palladio stated that:

“There are seven types of room that are the most beautiful and well proportioned and turn out better: they can be made circular [ritondo], though these are rare; or square [quadrate]; or their length will equal the diagonal of the square [quadrate] of the breadth; or a square [quadro] and a third; or a square and a half; or a square and two-thirds; or two squares” (Palladio 57).
Palladio’s seven preferred room length/width ratios

Although the *Quattro Libri* provides little evidence in how these ratios were selected by Palladio as preferred room ratios, many scholars believe that they have strong references to precedents of preferred ratios in treatises by Vitruvius and Alberti. However, the ratio of square and two-thirds (3:5) is a departure of Alberti’s strictly Pythagorean ratios (Padovan 35). This has led German art historian Rudolf Wittkower to link the selection of ratios to the development of music intervals in North Italian Renaissance music theory (Mitrovic 87). *Quattro Libri*

Wittkower studied the plans in Book II of the *Quattro Libri* and suggested that Palladio intentionally used certain room length/width ratios because they corresponded to musical intervals (83). In fact, his study identifies with the Renaissance belief that “mathematics and musical concord were basis of ideal proportion” (Rowe 8). The ratios illustrated by Wittkower were derived from intervals of the Pythagorean or Just tuning systems, which were both commonly used during the Renaissance (see Table x). Despite criticisms in the academic world with regards to this correlation, Wittkower’s study nonetheless provides a new perspective on the possible reasoning behind Palladio’s ratio selections, which has strong accordance to Renaissance belief and musical development (Padovan 234).
In the study “Harmonic Proportion and Palladio’s *Quattro Libri*” by Deborah Howard and Malcolm Longair, Palladio’s use of the ratios in his executed buildings were compared against his seven favorite room length/width proportions listed in the *Quattro Libri*. It was found that out of 153 length/width ratios stated on the plans, only 89 (55%) corresponded to Palladio’s preferred ratios. Since nearly half of the length/width ratios were not derived from his preferred list, it is important to look into other proportional systems in the Renaissance from which these 45% of ratios were based upon (Howard 130).

The ratios that fall under the 45% were all incommensurable, meaning they cannot be represented by whole numbers. This would present a controversy in the humanist ideal, as “Palladio’s conception of architecture, as indeed that of all Renaissance architects, is based on commensurability of ratios” (Wittkower 108). However, there must be other proportional systems in which these ratios can be represented that permit their acceptance and utilization by Renaissance Architects. Systems that have potentially derived these ratios include Triangulation, the Golden Section and Delian Cubes (Mitrovic 65).

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### Octave divisions of the Pythagorean and Just Tuning Scales

<table>
<thead>
<tr>
<th></th>
<th>CC</th>
<th>CD</th>
<th>CE</th>
<th>CF</th>
<th>CG</th>
<th>CA</th>
<th>CB</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagorean Scale</td>
<td>1/1</td>
<td>9/8</td>
<td>81/6</td>
<td>4</td>
<td>3/2</td>
<td>27/1</td>
<td>243/1</td>
<td>2/1</td>
</tr>
<tr>
<td>Just Tuning</td>
<td>1/1</td>
<td>9/8</td>
<td>5/4</td>
<td>4/3</td>
<td>3/2</td>
<td>5/3</td>
<td>15/8</td>
<td>2/1</td>
</tr>
</tbody>
</table>

*Red marks the ratios found in Palladio’s preferred list*
<table>
<thead>
<tr>
<th>Mathematical Concept</th>
<th>Ratio(s)</th>
<th>Applications</th>
<th>Controversy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangulation</td>
<td>( \sqrt{2} ) or 1.41421 (diagonal of square) ( \sqrt{3} ) or 1.73205, etc (diagonal of cube)</td>
<td>Atrio Toscano Villa Rotunda</td>
<td>Ratios cannot be expressed by whole numbers, which was contradictory to Renaissance humanist principles</td>
</tr>
<tr>
<td>Golden Section</td>
<td>( \frac{1+\sqrt{5}}{2} ) or 1.61803</td>
<td>Villa Godi Cornaro</td>
<td>Few Renaissance sources discussed Golden Section for its widespread application</td>
</tr>
<tr>
<td>Delian Cube</td>
<td>( \frac{3}{2} \sqrt{2} ) or 1.25992 (Ratio of edges of two cubes with volume ratios of 1:2) ( \frac{3}{2} \sqrt{2}^2 ) or 1.58740 (Ratio of surface areas of two cubes with volume ratios of 1:2)</td>
<td>Villa Capra Villa Della Torre</td>
<td>No known Renaissance precedent for using Delian Cube in proportional procedure</td>
</tr>
</tbody>
</table>

Potential mathematical systems used for deriving incommensurable ratios

Proportions in Room Height

Palladio has also employed the proportional relationships other architects had harnessed for two dimensions of façade or three dimensions of a single room in establishing his set of rules to govern room height (Padovan 235). The rules differ depending on whether the room has a flat ceiling or vaulted ceiling, and they are outlined in the following table.

<table>
<thead>
<tr>
<th>Room Configuration</th>
<th>Height Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Ceiling</td>
<td>Equals the width of room</td>
</tr>
<tr>
<td>Vaulted Ceiling</td>
<td>Arithmetic, geometric or harmonic mean of room’s length and width</td>
</tr>
<tr>
<td>Vaulted Ceiling - square room</td>
<td>4/3 of the width</td>
</tr>
</tbody>
</table>

Palladio’s proportional rules for room height (Padovan 233)

In order to truly integrate individual rooms into a unifying building, Palladio established the Condition on the Concordance of Heights (CCH) Rule, to relate the proportions of one room to another. This ensures that while all rooms on the same
floor have the same height, the resulting room length/width ratios would conform to his preferred list.

**Proportions in the Villa Rotunda**

The Villa Rotunda is selected as a model to be analyzed, to understand how Palladio applied his rules of proportions to his executed works. In this exercise, the length, width and height of each room is tabulated and compared against the preferred proportions used by Palladio that were previously identified. The tabulated results, as well as room location diagrams, are shown in the table in Appendix A.

From the tabulation and diagrams, it is evident that most of the rooms with length/width ratios conforming to Palladio’s seven preferred ratios are located on the top floor. Rooms with conformity to the height rule can only be identified on the ground floor. None of the rooms in the Villa Rotunda conforms to both the preferred length/width ratios and height rule.

Therefore, it can be inferred that while Palladio has emphasized the significance of the seven preferred length/width ratios in the *Quattro Libri*, he did not follow them rigorously in his design of the Villa. Furthermore, Palladio may have applied the room height rules only for rooms at the ground floor of the villa, which are interior spaces that can be experienced by most visitors of the building.

However, a closer examination of the ratios at the ground floor would reveal deeper proportional relationship between the Rooms 1-c and 1-d. Padovan speculates that although the length/width ratios for Rooms 1-c and 1-d are not on Palladio’s preferred list of length/width ratios, their length/width proportions can be expressed in the forms of incommensurable ratios of \((\sqrt{3}+1):2\) and \(\sqrt{3}:1\), respectively (236). The ratio of \(\sqrt{3}:1\) also exists between the side of the equilateral triangle inscribed in the central rotunda and the rotunda’s radius. Therefore, the plan exhibits a strong theme of \(\sqrt{3}\). According to Padovan, this theme may be used by Palladio intentionally to symbolize the holy trinity, as the client of the villa was a retired Monsignore (236).
Speculative Theme of $\sqrt{3}$ as expressed by room proportions in the Villa Rotunda

*Effects of Proportion on Illumination by Natural Lighting*

From a comprehensive study of the design of Villa Rotunda covering the aspects of entry, illumination, room function and structure (see Appendix B), the topic of interior space illumination is selected for further examination. A lighting experiment was conducted to examine how the changes in the room’s length/width ratio, height and the ceiling type would affect the illumination of the interior space.

It is hypothesized that the rules of proportion applied by Andreas Palladio in the design of the Villa’s rooms were not only mere reflections of the mathematical ideal, but they were established to optimize the human experience of natural lighting.
The two rooms at the southeast corner on the ground floor were modeled for the study. Eighteen study models were built with varying room depth, ceiling height and ceiling type. The models with varied heights and room depth do not follow any of the proportional rules set by Palladio.

A model base was made on which the sun directions and elevations were marked for 5 time points each for winter solstice and summer solstice. The 18 models were
placed on the base individually, at which a picture was taken for each of the ten sun direction setting.

Appendix C displays all 180 photos taken in the experiment, classified by roof type and by season. It is evident that the contrast of brightness between the larger and smaller room are greater in summer than in winter, and that the room is more illuminated in winter than in summer. No other trends are observed from the set of photos.

To better understand the relationship between proportion and illumination, the 12:00 columns of the winter and summer solstices were extracted and reconfigured in 3x3 grid for further comparison. The comparison slides and the observations are included in Appendices D and E.
By comparing the light study images, it is observed that,

1. The vault can make the light gentler, and it works best in the original-proportioned room
2. The level of illumination in the room increases as room-depth decreases, due to close proximity of wall surface to the window to reflect and disperse light in shallower rooms
3. The level of illumination in the heightened room and shallow room is almost the same. And it is brighter than the original proportioned room

From these findings, it is evident that the changes in room proportions in the selected rooms do have a strong impact on the illumination of interior space by natural light. As the vaulted ceiling performs best in harmonizing illumination in the original proportioned model, it may be speculated that Palladio has intentionally fine-tuned the room’s proportion with the placement of the vaulted ceiling to enable harmony illumination in the room. However, further studies need to be conducted to other rooms in the building to determine the consistency of the effect of room proportion on interior illumination.

Conclusion

An era that is in pursuant of the humanistic ideal, “the Renaissance was an age that delighted in number symbolism and in mathematical games, analogies and conceits, whether in musical composition, the making of verse, or architecture.” (Padovan 236). As a Renaissance architect, Palladio exemplified attentiveness in his use of “ideal proportions” in his buildings, and he developed rules for establishing the most beautiful room proportions in the Quattro Libri.

However, a closer examination of his buildings, such as the Villa Rotunda, reveals that he did not apply these rules rigorously in his building design, as he needed to fulfill other design requirements, such as structure, function, site context and illumination. From the light study of the Villa Rotunda, it can be inferred that Palladio did place consideration in room proportions to influence illumination of the interior space; however, the extent to which relied on proportions to achieve this deserves further investigation.
Semenzato, C. (1968) The Rotunda of Andrea Palladio